

STAT 3375Q: Introduction to Mathematical Statistics I
Spring 2024

Quiz 4 Review Exercises

Quiz Date: 20 March 2024

INSTRUCTIONS:

- The quiz is consist of only one problem taken from this list of five problems.
- This is a closed book, closed notes, closed laptop/computer quiz.
- The duration of the quiz is 15 minutes.
- The material for the quiz is Lectures 10-14 and the homework exercises.
- A calculator is not necessary. You can keep your final answers as fractions in the simplest form.
- To merit partial points, make sure to justify/explain your thoughts and solutions, using notation and terminology properly, and clearly defining any events and random variables that you use.
- Do not be late. The quiz will start at exactly 4:40pm and end at 4:55pm.

Problem 1

Suppose the grades on this quiz is normally distributed with a mean score of 70 points and standard deviation of 10 points. Furthermore, suppose I decide to give the top 10% a bonus of 5 points. What should be the cutoff score to merit the bonus points?

Solution:

Problem 2

Let X have MGF given by

$$m(t) = \frac{1}{3}e^t + \frac{2}{3}e^{2t}, \quad t \in \mathbb{R}.$$

- a) What is the distribution of X ?
- b) Find the expected value and variance of X .

Solution:

Problem 3

Verify that the standard normal PDF

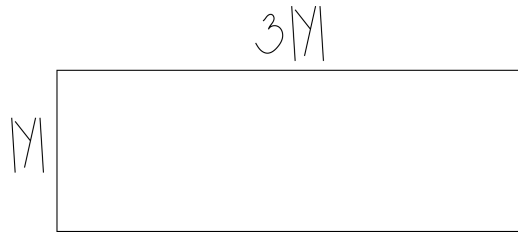
$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, \quad -\infty < z < \infty.$$

is a valid PDF.

Solution:

Problem 4

Suppose $Y \sim \mathcal{N}(\mu, \sigma^2)$. Find the expected value of the area of the rectangle below.



Solution:

Problem 5

Suppose that X has the Gamma distribution with parameters α and β . Let c be a positive constant. Show that cX has the Gamma distribution with parameters α and $c\beta$.

Solution: