

STAT 3375Q: Introduction to Mathematical Statistics I
Spring 2024

Week 10 Homework Exercises

Discussion Date: 22 March 2024

Problem 4.139

The MGF of a normally distributed random variable Y with mean μ and variance σ^2 is

$$m(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}.$$

Derive the MGF of $X = -3Y + 4$. What is the distribution of X ? Why?

Solution:

Problem 4.141

If $\theta_1 < \theta_2$, derive the MGF of a uniform random variable on the interval (θ_1, θ_2) .

Solution:

Problem 4.142

Suppose that Y is uniformly distributed on the interval $(0, 1)$ and that $a > 0$ is a constant.

- a) Give the MGF of Y .
- b) Derive the MGF of $W = aY$. What is the distribution of W ? Why?
- c) Derive the MGF of $W = -aY$. What is the distribution of W ? Why?
- d) If b is a fixed constant, derive the MGF of $V = aY + b$. What is the distribution of V ? Why?

Solution:

Problem 4.143

Differentiate the MGF of a Gamma random variable to find the mean and variance of the Gamma distribution.

Solution:

Problem 4.181

Suppose that Y is a normally distributed random variable with mean μ and variance σ^2 . Find the MGF, mean, and variance of

$$Z = \frac{Y - \mu}{\sigma}.$$

What is the distribution of Z ? Why?

Solution: