

STAT 3375Q: Introduction to Mathematical Statistics I
Spring 2024

Week 2 Homework Exercises

Discussion Date: 26 January, 2024

Problem 2.15

An oil prospecting firm hits oil or gas on 10% of its drilling. If the firm drills two wells, the four possible simple events and three of their associated probabilities are as given in the accompanying table. Find the probability that the company will hit oil or gas

Simple Event	Outcome of First Drilling	Outcome of Second Drilling	Probability
E_1	Hit (oil or gas)	Hit (oil or gas)	0.01
E_2	Hit	Miss	?
E_3	Miss	Hit	0.09
E_4	Miss	Miss	0.81

- a) on the first drilling and miss on the second.
- b) on at least one of the two drillings.

Solution:

Problem 2.17

Hydraulic landing assemblies coming from an aircraft rework facility are each inspected for defects. Historical records indicate that 8% have defects in shafts only, 6% have defects in bushings only, and 2% have defects in both shafts and bushings. One of the hydraulic assemblies is selected randomly. What is the probability that the assembly has

- a) a bushing defect?
- b) a shaft or bushing defect?
- c) exactly one of the two types of defects?
- d) neither type of defect?

Solution:

Problem 2.19

A business office orders paper supplies from one of three vendors, V_1 , V_2 , or V_3 . Orders are to be placed on two successive days, one order per day. Thus, (V_2, V_3) might denote that vendor V_2 gets the order on the first day and vendor V_3 gets the order on the second day.

- a) List the sample points in this experiment of ordering paper on two successive days.
- b) Assume the vendors are selected at random each day and assign a probability to each sample point.
- c) Let A denote the event that the same vendor gets both orders and B the event that V_2 gets at least one order. Find $P(A)$, $P(B)$, $P(A \cup B)$, and $P(A \cap B)$ by summing the probabilities of the sample points in these events.

Solution:

Problem 2.21

If A and B are events, use the result derived in Exercise 2.5(a) and the Axioms in Definition 2.6 to prove that

$$P(A) = P(A \cap B) + P(A \cap \bar{B})$$

Solution:

Problem 2.23

If A and B are events and $B \subset A$, why is it “obvious” that $P(B) \leq P(A)$.

Solution:

Problem 2.75

Cards are dealt, one at a time, from a standard 52-card deck.

- a. If the first 2 cards are both spades, what is the probability that the next 3 cards are also spades?
- b. If the first 3 cards are all spades, what is the probability that the next 2 cards are also spades?
- c. If the first 4 cards are all spades, what is the probability that the next card is also a spade?

Solution:

Problem 2.77

A study of the posttreatment behavior of a large number of drug abusers suggests that the likelihood of conviction within a two-year period after treatment may depend upon the offenders education. The proportions of the total number of cases falling in four education–conviction categories are shown in the following table:

Education	Status within 2 Years after Treatment		
	Convicted	Not-convicted	Total
10 years or more	0.10	0.30	0.40
9 years or less	0.27	0.33	.60
Total	0.37	0.63	1.00

Suppose that a single offender is selected from the treatment program. Define the events:

A: The offender has 10 or more years of education.

B: The offender is convicted within two years after completion of treatment.

Find the following:

- $P(A)$
- $P(B)$
- $P(A \cap B)$
- $P(A \cup B)$
- $P(\bar{A})$
- $P(\overline{A \cup B})$
- $P(\overline{A \cap B})$
- $P(A|B)$
- $P(B|A)$

Solution: