Solution for Week2 Discussion Session

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2.15

All the four events are mutually exclusive and exhaustive. Hence, $P(S) = P(E_1)+P(E_2)+P(E_3)+P(E_4) = 1$
(a) From the given table we have, $P(E_2) = 1 - (0.01 + 0.09 + 0.81) = 0.09$
(b) Probability that company will hit at least one of the two drillings i.e.
$P(\text{at least one hit}) = P(E_1) + P(E_2) + P(E_3) = 0.01 + 0.09 + 0.09 = 0.19$

2.17

Given that, 8% have defects in shafts only i.e. $P(\text{shafts defect}) = 0.08$, 6% have defects in bushings only i.e. $P(\text{bushings defect}) = 0.06$, and 2% have defects in both shafts and bushings i.e. $P(\text{shafts} \cap \text{bushings}) = 0.02$
(a) The probability that the assembly has a bushing defect is:
$P(\text{bushings defect}) + P(\text{shafts} \cap \text{bushings}) = 0.06 + 0.02 = 0.08$
(b) The probability that the assembly has a shaft or bushing effect is:
$P(\text{shafts}\cup\text{bushings}) = P(\text{bushings defect})+P(\text{shafts defect})+P(\text{shafts}\cap\text{bushings}) = 0.06+0.08+0.02 = 0.16$
(c) $P(\text{exactly one defect}) = .06 + .08 = .14$
(d) $P(\text{neither defect}) = 1 – P(\text{shafts} \cup \text{bushings}) = 1 – 0.16 = 0.84$

2.19

(a) The sample points in this experiment of ordering paper on two successive days are listed below:
$\{(V_1, V_1), (V_1, V_2), (V_1, V_3), (V_2, V_1), (V_2, V_2), (V_2, V_3), (V_3, V_1), (V_3, V_2), (V_3, V_3)\}$
(b) The vendors are selected at random each day and the sample points are equally likely. Hence the probability is 1/9 for each sample point.
(c) A denote the event that the same vendor gets both orders and B the event that V_2 gets at least one order. Therefore,
$A = \{(V_1, V_1), (V_2, V_2), (V_3, V_3)\}$
$B = \{(V_1, V_2), (V_2, V_1), (V_2, V_2), (V_2, V_3), (V_3, V_2)\}$
So, P(A) = 1/3, P(B) = 5/9, P(A ∪ B) = 7/9, P(A ∩ B) = 1/9.

2.21

From 2.5 we have, $P(A) = P((A \cap B) \cup (A \cap \overline{B}))$ And as we have shown that $(A \cap B)$ and $(A \cap \overline{B})$ are mutually exclusive i.e intersection is ϕ.
Hence, we can write:
$P(A) = P((A \cap B) \cup (A \cap \overline{B})) = P(A \cap B) + P(A \cap \overline{B})$

2.23

Given that A and B are events and $B \subset A$. Hence, all elements in B are in A,so that when B occurs, A must also occur. However, it is possible for A to occur and B not to occur. All elements in B are present in A. Still A can have extra elements that are not in B. As the probability of a event is always positive and A contains extra elements, so it is obvious that $P(B) \leq P(A)$.

2.75

Cards are dealt, one at a time, from a standard 52-card deck.
(a) The first 2 cards are both spades. We need to find the probability that the next 3 cards are also spades. Given the first two cards drawn are spades, there are 11 spades left in the deck. Hence, the probability will be $\frac{11_{C_3}}{50_{C_3}} = 0.0084$.
(b) Given the first three cards drawn are spades, there are 10 spades left in the deck. Thus, the probability is $\frac{10_{C_2}}{49_{C_2}} = 0.0383$.
(c) Given the first four cards drawn are spades, there are 9 spades left in the deck. Thus, the probability is $\frac{9_{C_1}}{48_{C_1}} = 0.1875$.

2.77

Suppose we are selecting a single offender from the treatment program. And there are two sets defined as follows:
A: The offender has 10 or more years of education.
B: The offender is convicted within two years after completion of treatment.
(a) $P(A) = 0.40$
(b) $P(B) = 0.37$
(c) $P(A \cap B) = 0.10$
(d) $P(A \cup B) = 0.40 + 0.37–0.10 = 0.67$
(e) $P(\overline{A}) = 0.60$
(f) $P(\overline{A \cup B}) = 1 - 0.67 = 0.33$
(g) $P(\overline{A \cap B}) = 1 - 0.10 = 0.90$
(h) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.10}{0.37} = 0.27$
(i) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.10}{0.40} = 0.25$