UCONN
UNIVERSITY OF CONNECTICUT

Department of Statistics
College of Liberal Arts & Sciences

STAT 3375Q: Introduction to Mathematical Statistics I

Spring 2024

Final Simulation Exam Date: 29 April 2024

INSTRUCTIONS:

- There are 7 problems in this exam. Pick ONLY 5 problems to answer. Indicate your 5 chosen problems by circling the numbers on the table below. Answering more than 5 problems will NOT merit additional points.
- You are allowed ONE formula sheet which you will SUBMIT along with this exam sheet. Put all other items away such as books, notes, phones, laptops, and other electronic devices.
- You have 2 hours to complete the exam. Time remaining will be flashed on the screen and will be updated every 10 minutes.
- A calculator is not necessary. You can keep your final answers as fractions in the simplest form.
- Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- To merit partial points, make sure to justify/explain your thoughts and solutions, using notations and terminologies properly, and clearly defining any events, random variables, parameters, and distributions that you used.
- Mysterious or unsupported answers will not receive full credit. A correct answer, unsupported by calculations, explanations, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Allocated Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
Total	100	

NAME: ______________________________

Problem 1
Suppose that a random variable X can take each of the five values $-2, -1, 0, 1, 2$ with equal probability. Let $Y = |X| - X$.

a) Find the distribution Y. *(8 points)*

b) Compute the mean of Y. *(6 points)*

c) Compute the variance of Y. *(6 points)*

Solution:

Problem 2
Let X be a uniform RV over the interval $[-4, 6]$. Suppose we have another RV Y also uniform over the interval $[a, 4a]$.

a) Find the mean of X. *(5 points)*

b) Find $P(X \leq 2.4)$. *(5 points)*

c) Find $P(-3 \leq X - 2 \leq 3)$. *(5 points)*

d) Given that $P(X \leq 1) = P(Y \leq 1)$, find the value of a. *(5 points)*

Solution:

Problem 3
Let X and Y be continuous random variables with joint PDF

$$f(x,y) = \begin{cases} 8xy, & \text{if } 0 < y < x, 0 < x < 1; \\ 0, & \text{elsewhere.} \end{cases}.$$

a) Find $P(X > 1/2)$. *(5 points)*

b) Find $P(Y < 3/5, X > 1/2)$. *(5 points)*

c) Find $P(Y < 3/5 | X > 1/2)$. *(5 points)*

d) Are X and Y independent? *(5 points)*

Solution:

Problem 4

a) Let X be a Gaussian random variable with $\mu = 10$ and $\sigma^2 = 36$. Find $P(4 < X < 16)$. *(6 points)*

b) Let X be a Gaussian random variable with $\mu = 5$. If $P(X > 9) = 0.2$, compute $V(X)$. *(7 points)*

c) Let X be a Gaussian random variable with $\mu = 12$ and $\sigma^2 = 4$. Find the value of c such that $P(X > c) = 0.10$. *(7 points)*

Solution:

Problem 5
Let X and Y be random variables such that

$$E(X) = 2, \quad E(Y) = 1, \quad E(X^2) = 5, \quad E(Y^2) = 10, \quad E(XY) = 1.$$

a) Find $\text{Cov}(X, Y)$. *(4 points)*

b) Find $V(X)$. *(4 points)*

c) Find $V(Y)$. *(4 points)*

d) Find $\text{Corr}(X, Y)$. *(4 points)*

e) Find a number c so that X and $X + cY$ are uncorrelated. *(4 points)*

Solution:

Problem 6

Suppose $X_1, X_2, \ldots, X_n \stackrel{iid}{\sim} \text{Exp}(\beta)$. Define $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ as the sample mean.

a) Find the distribution of the sample mean. *(8 points)*

b) Compute the mean of the sample mean. *(6 points)*

c) Compute the variance of the sample mean. *(6 points)*

Solution:

Problem 7
Let X have the following PDF:

$$f_X(x) = \begin{cases} \frac{x^2}{9}, & \text{if } 0 < x < 3; \\ 0, & \text{elsewhere.} \end{cases}$$

Find the PDF of $Y = X^3$ using the Jacobian method. *(20 points)*

Solution: