STAT 3375Q: Introduction to Mathematical Statistics I

Lecture 8: Continuous Random Variables

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Outline

Midterm 1 Solutions

Problem 1

A pair of events A and B cannot be simultaneously mutually exclusive and independent. Prove that if $P(A) > 0$ and $P(B) > 0$, then:

(a) If A and B are mutually exclusive, they cannot be independent. *(10 points)*

(b) If A and B are independent, they cannot be mutually exclusive. *(10 points)*

Solution:

(a) Given: A and B are mutually exclusive.
$\Rightarrow A \cap B = \emptyset$.
$\Rightarrow P(A \cap B) = 0$.
If A and B are independent (p argument), then $P(A) = 0$ or $P(B) = 0$ (q argument) since $P(A \cap B) = 0$ and $P(A \cap B) = P(A)P(B)$.
Since it is given that $P(A) > 0$ and $P(B) > 0$ (-q), then A and B cannot be independent (-p).

(b) Given: A and B are independent.
$\Rightarrow P(A \cap B) = P(A)P(B)$.
$\Rightarrow P(A \cap B) > 0$ since $P(A) > 0$ and $P(B) > 0$.
$\Rightarrow P(A \cap B) \neq 0$.
$\Rightarrow A$ and B are not mutually exclusive.

□

Problem 2

Prove each of the following statements. (Assume that any conditioning event has positive probability.)

- **a** If $P(B) = 1$, then $P(A|B) = P(A)$ for any A. *(5 points)*
- **b** If $A \subset B$, then $P(B|A) = 1$ and $P(A|B) = \frac{P(A)}{P(B)}$. *(5 points)*
- **c** If A and B are mutually exclusive, then

$$P(A|A \cup B) = \frac{P(A)}{P(A) + P(B)}. \qquad \textit{(5 points)}$$

- **d** $P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$. *(5 points)*

Solution:

- **a** $P(A|B) = \frac{P(A \cap B)}{P(B)} = P(A \cap B)$.
  Let B and $\bar{B}$ be a partition of the sample space S, i.e., $S = B \cup \bar{B}$ and $B \cap \bar{B} = \emptyset$.
  By law of total probability, $P(A) = P(A \cap B) + P(A \cap \bar{B})$.
  Since $(A \cap \bar{B}) \subset \bar{B}$ and $P(\bar{B}) = 1 - P(B) = 1 - 1 = 0$, $P(A \cap \bar{B}) = 0$.
  Hence, $P(A) = P(A \cap B)$.
  Therefore, $P(A|B) = P(A)$.

Problem 2

Prove each of the following statements. (Assume that any conditioning event has positive probability.)

a. If $P(B) = 1$, then $P(A|B) = P(A)$ for any A. *(5 points)*
b. If $A \subset B$, then $P(B|A) = 1$ and $P(A|B) = \frac{P(A)}{P(B)}$. *(5 points)*
c. If A and B are mutually exclusive, then

$$P(A|A \cup B) = \frac{P(A)}{P(A) + P(B)}. \qquad \textit{(5 points)}$$

d. $P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$. *(5 points)*

Solution:

b. If $A \subset B$, then $A \cap B = A$.
$\Rightarrow P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A)}{P(A)} = 1$. Also, $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$.
c. If A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$

$$P(A|A \cup B) = \frac{P\{A \cap (A \cup B)\}}{P(A \cup B)} = \frac{P(A)}{P(A) + P(B)}.$$

d. $P(A \cap B \cap C) = P(A \cap (B \cap C)) = P(A|B \cap C)P(B \cap C) = P(A|B \cap C)P(B|C)P(C)$.

Problem 3

Cards are dealt, one at a time, from a standard 52-card deck.

- (a) If the first 2 cards are both spades, what is the probability that the next 3 cards are also spades? *(6 points)*
- (b) If the first 3 cards are all spades, what is the probability that the next 2 cards are also spades? *(7 points)*
- (c) If the first 4 cards are all spades, what is the probability that the next card is also a spade? *(7 points)*

Solution:

- (a) Approach 1.

  $$P(S_3S_4S_5|S_1S_2) = \frac{P(S_3 \cap S_4 \cap S_5 \cap S_1 \cap S_2)}{P(S_1 \cap S_2)}$$
  $$= \frac{(\frac{13}{52})(\frac{12}{51})(\frac{11}{50})(\frac{10}{49})(\frac{9}{48})}{(\frac{13}{52})(\frac{12}{51})} = 0.084$$

  Approach 2. 11 spades left, 50 cards left.

  $$P(S_3S_4S_5|S_1S_2) = \frac{\binom{11}{3}}{\binom{50}{3}} = 0.084.$$

Problem 3

Cards are dealt, one at a time, from a standard 52-card deck.

- **a** If the first 2 cards are both spades, what is the probability that the next 3 cards are also spades? *(6 points)*
- **b** If the first 3 cards are all spades, what is the probability that the next 2 cards are also spades? *(7 points)*
- **c** If the first 4 cards are all spades, what is the probability that the next card is also a spade? *(7 points)*

Solution:

- **b** Approach 1.

$$P(S_4 S_5 | S_1 S_2 S_3) = \frac{P(S_3 \cap S_4 \cap S_5 \cap S_1 \cap S_2)}{P(S_1 \cap S_2 \cap S_3)}$$

$$= \frac{(\frac{13}{52})(\frac{12}{51})(\frac{11}{50})(\frac{10}{49})(\frac{9}{48})}{(\frac{13}{52})(\frac{12}{51})(\frac{11}{50})} = 0.383$$

Approach 2. 10 spades left, 49 cards left.

$$P(S_4 S_5 | S_1 S_2 S_3) = \frac{\binom{10}{2}}{\binom{49}{2}} = 0.383.$$

Problem 3

Cards are dealt, one at a time, from a standard 52-card deck.

- (a) If the first 2 cards are both spades, what is the probability that the next 3 cards are also spades? *(6 points)*
- (b) If the first 3 cards are all spades, what is the probability that the next 2 cards are also spades? *(7 points)*
- (c) If the first 4 cards are all spades, what is the probability that the next card is also a spade? *(7 points)*

Solution:

- (c) Approach 1.

$$P(S_5|S_1S_2S_3S_4) = \frac{P(S_3 \cap S_4 \cap S_5 \cap S_1 \cap S_2)}{P(S_1 \cap S_2 \cap S_3 \cap S_4)}$$

$$= \frac{(\frac{13}{52})(\frac{12}{51})(\frac{11}{50})(\frac{10}{49})(\frac{9}{48})}{(\frac{13}{52})(\frac{12}{51})(\frac{11}{50})(\frac{10}{49})} = 0.1875$$

Approach 2. 9 spades left, 48 cards left.

$$P(S_5|S_1S_2S_3S_4) = \frac{\binom{9}{1}}{\binom{48}{1}} = 0.1875.$$

Problem 4

Suppose that 5% of men (M) and 0.25% of women (F) are color blind. A person is chosen at random and that person is color blind (CB). What is the probability that the person is male? (Assume males and females to be in equal numbers.) *(20 points)*

Solution:

Using Bayes' rule, we have

$$\begin{aligned} P(M|CB) &= \frac{P(CB|M)P(M)}{P(CB|M)P(M) + P(CB|F)P(F)} \\ &= \frac{(0.05)(0.5)}{(0.05)(0.5) + (0.0025)(0.5)} \\ &= 0.9524. \end{aligned}$$

□

Problem 5

Suppose X has the geometric PMF $p(x) = \frac{1}{3}\left(\frac{2}{3}\right)^x$, $x = 0, 1, 2, \ldots$. Determine the probability distribution of $Y = \frac{X}{X+1}$. Note that both X and Y are discrete random variables. To specify the probability distribution means to specify its PMF. *(20 points)*

Solution:

$$
\begin{aligned}
P(Y = y) &= P\left(\frac{X}{X+1} = y\right) \\
&= P\left(X = Xy + y\right) \\
&= P\left(X - Xy = y\right) \\
&= P\left(X(1-y) = y\right) \\
&= P\left(X = \frac{y}{1-y}\right) \\
&= p(\frac{y}{1-y}) \\
&= \frac{1}{3}\left(\frac{2}{3}\right)^{\frac{y}{1-y}},
\end{aligned}
$$

where $y = 0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{x}{x+1}, \ldots$.

Problem 6

Suppose the random variable X has a Poisson distribution such that $P(X = 1) = P(X = 2)$.

a. Find $P(X = 4)$. *(10 points)*

b. Find $P(X \geq 4 | X \geq 2)$. *(10 points)*

Solution: Recall the Poisson PMF for a random variable Y: $p(y) = \frac{\lambda^y}{y!} e^{-\lambda}$.

a.

$$\begin{aligned} P(X = 1) &= P(X = 2) \\ p(1) &= p(2) \\ \frac{\lambda e^{-\lambda}}{1!} &= \frac{\lambda^2 e^{-\lambda}}{2!} \\ \lambda &= \frac{\lambda^2}{2} \\ \lambda &= 2. \end{aligned}$$

Therefore, $P(X = 4) = p(4) = \frac{2^4 e^{-2}}{4!} = \frac{2}{3} e^{-2}$.

Problem 6

Suppose the random variable X has a Poisson distribution such that $P(X=1)=P(X=2)$.

- (a) Find $P(X=4)$. *(10 points)*
- (b) Find $P(X \geq 4 | X \geq 2)$. *(10 points)*

Solution: Recall the Poisson PMF for a random variable Y: $p(y) = \frac{\lambda^y}{y!} e^{-\lambda}$.

- (b)

$$
\begin{aligned}
P(X \geq 4 | X \geq 2) &= \frac{P(X \geq 4 \text{ and } X \geq 2)}{P(X \geq 2)} = \frac{P(X \geq 4)}{P(X \geq 2)} \\
&= \frac{1 - \{p(0) + p(1) + p(2) + p(3)\}}{1 - \{p(0) + p(1)\}} \\
&= \frac{1 - \{\frac{2^0 e^{-2}}{0!} + \frac{2^1 e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!} + \frac{2^3 e^{-2}}{3!}\}}{1 - \{\frac{2^0 e^{-2}}{0!} + \frac{2^1 e^{-2}}{1!}\}} \\
&= \frac{1 - e^{-2}\{\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!}\}}{1 - e^{-2}\{\frac{2^0}{0!} + \frac{2^1}{1!}\}} \\
&= \frac{1 - e^{-2}(1 + 2 + 2 + \frac{4}{3})}{1 - e^{-2}(1+2)} = \frac{1 - e^{-2}(\frac{19}{3})}{1 - e^{-2}(3)} \approx 0.24.
\end{aligned}
$$

□

Problem 7

Let Y be the number of successes throughout n independent repetitions of a random experiment having the probability of success $p = \frac{1}{4}$. Determine the smallest value of n so that $P(Y \geq 1) \geq 0.70$. *(20 points)*

Solution:

Recall the binomial PMF for a random variable Y: $p(y) = \binom{n}{y} p^y (1-p)^{n-y}$, $y = 0, 1, \ldots, n$.

Note that it is easier to solve the probabilities of the complement.

$$\begin{aligned} P(Y \geq 1) &= 1 - P(Y = 0) \\ &= 1 - \binom{n}{0} \left(\frac{1}{4}\right)^0 \left(1 - \frac{1}{4}\right)^{n-0} \\ &= 1 - \left(\frac{3}{4}\right)^n. \end{aligned}$$

We need to find n such that

$1 - \left(\frac{3}{4}\right)^n \geq 0.7 \Rightarrow 0.3 \geq \left(\frac{3}{4}\right)^n \Rightarrow \log(0.3) \geq n \log\left(\frac{3}{4}\right) \Rightarrow n > \frac{\log(0.3)}{\log\left(\frac{3}{4}\right)} = 4.19.$

Therefore, n must be 5 so that $P(Y \geq 1) \geq 0.70$.

Previously...

Discrete Random Variables

- Random Variable: is a function that maps outcomes into real numbers.
- Discrete Random Variables: random variables whose values take only a finite or countably infinite number of possible values.
- Expected Value of Discrete Random Variable Y:

$$\mu = E(Y) = \sum_{y} yp(y).$$

- Variance of Discrete Random Variable Y:

$$\sigma^2 = V(Y) = E\{(Y-\mu)^2\} = \sum_{y}(y-\mu)^2 p(y) = E(Y^2) - \mu^2.$$

- Standard Deviation of Discrete Random Variable Y:

$$\sigma = \sqrt{V(Y)}.$$

Special Discrete Distributions

	Bernoulli	**Binomial**	**Poisson**
Usage	success or failure?	count num. of success in n trials	count num. of times an event occurs w/n specified time interval
Parameters	p (prob. success)	n (num. of trials), p (prob. success)	λ (average or expected rate of occurence)
Notation	$Y \sim Be(p)$	$Y \sim B(n, p)$	$Y \sim Poi(\lambda)$
PMF	$p(y) = p^y(1-p)^{1-y}$ $y = 0, 1$	$p(y) = \binom{n}{y} p^y(1-p)^{n-y}$ $y = 0, 1, 2, \ldots, n$	$p(y) = \frac{\lambda^y}{y!}e^{-\lambda}$ $y = 0, 1, 2, \ldots$
Mean	p	np	λ
Variance	$p(1-p)$	$np(1-p)$	λ

(cont'd next slide...)

Special Discrete Distributions (cont'd)

	Geometric Type 1	**Negative Binomial** Type 1	**Hypergeometric**
Usage	count num. of trials to get first success	count num. of trials to get rth success	count num. of success w/o replacement
Parameters	p (prob. success)	r (num. of success) p (prob. success)	N (population size) r (num. of success in population) n (sample size)
Notation	$Y \sim Geo(p)$	$Y \sim NB(r, p)$	$Y \sim Hyper(N, r, n)$
PMF	$p(y) = (1-p)^{y-1}p$ $y = 1, 2, 3, \ldots$	$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r}$ $y = r, r+1, r+2, \ldots$	$p(y) = \dfrac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$ $y = 0, 1, 2, \ldots, n,$ $y \leq r$ and $n - y \leq N - r$
Mean	$\frac{1}{p}$	$\frac{r}{p}$	$\frac{nr}{N}$
Variance	$\frac{1-p}{p^2}$	$\frac{r(1-p)}{p^2}$	$n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$

The Cumulative Distribution Function

The Cumulative Distribution Function

Definition 4.1: The Cumulative Distribution Function (CDF)

Let Y denote any random variable (discrete or continuous). The *cumulative distribution function* of Y, denoted by $F(y)$, is such that $F(y) = P(Y \leq y)$ for $-\infty < y < \infty$.

- cumulative: increasing or growing by accumulation or successive additions
- CDF: essentially tells us what is the accumulated probability from $-\infty$ up until the value y.
- Even if Y may not take all real numbers, the CDF is defined for all $y \in \mathbb{R}$.

Note: The nature of the CDF of the random variable determines whether it is continuous or discrete.

The Cumulative Distribution Function

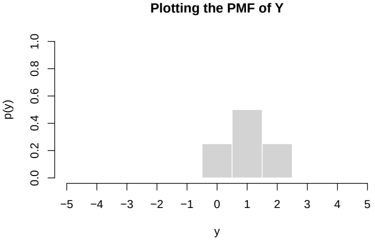
Example:

Suppose that $Y \sim B(n, p)$ with $n = 2$ and $p = 1/2$. Find $F(y)$.

Solution:

The PMF of Y is $p(y) = \binom{2}{y} \left(\frac{1}{2}\right)^y \left(\frac{1}{2}\right)^{2-y}$, $y = 0, 1, 2$. $p(y) = \binom{n}{y} p^y (1-p)^{n-y}$, $y = 0, 1, \ldots, n$

y	$p(y)$
0	1/4
1	1/2
2	1/4



y	$F(y)$
-5	0
-0.5	0
0	$1/4$
0.5	$1/4$
0.9	$1/4$
1	$1/4 + 1/2 = 3/4$
1.9	$1/4 + 1/2 = 3/4$
2	$1/4 + 1/2 + 1/4 = 1$
3	$1/4 + 1/2 + 1/4 = 1$

The CDF of Y is $F(y) = P(Y \leq y) = \begin{cases} 0, & \text{for } y < 0, \\ 1/4, & \text{for } 0 \leq y < 1, \\ 3/4, & \text{for } 1 \leq y < 2, \\ 1, & \text{for } y \geq 2. \end{cases}$

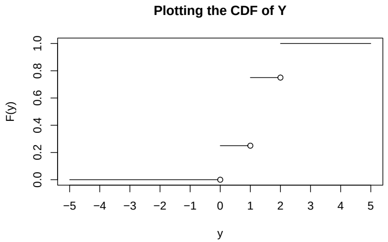
The Cumulative Distribution Function

Example:

Suppose that $Y \sim B(n, p)$ with $n = 2$ and $p = 1/2$. Find $F(y)$.

Solution:

The CDF of Y is
$$F(y) = P(Y \le y) = \begin{cases} 0, & \text{for } y < 0, \\ 1/4, & \text{for } 0 \le y < 1, \\ 3/4, & \text{for } 1 \le y < 2, \\ 1, & \text{for } y \ge 2. \end{cases}$$



- The graph of the CDF is a step function. The value of the CDF does not change between integers.
- This is the case for all discrete random variables.
- The value of the CDF jumps at the possible values of the random variable.
- The size of the jump is given by the value of the PMF at that possible value of the random variable.

The Cumulative Distribution Function

Theorem 4.1: Properties of the CDF

If $F(y)$ is a distribution function, then

1. $F(-\infty) = \lim_{y \to -\infty} F(y) = 0.$
2. $F(\infty) = \lim_{y \to \infty} F(y) = 1.$
3. $F(y)$ is a nondecreasing function of y.

   If y_1 and y_2 are any values such that $y_1 < y_2$, then $F(y_1) \leq F(y_2)$.

Proof:

1. $F(-\infty) = P(Y \leq -\infty) = P(\emptyset) = 0.$
2. $F(\infty) = P(Y \leq \infty) = P(S) = 1.$ S here is the sample space.
3. If $y_1 < y_2$, then the event $Y < y_1$ is a sub-set of the event $Y < y_2$. Then by the monotonicity property of sets,

$$F(y_1) = P(Y < y_1) \leq P(Y < y_2) = F(y_2).$$

The Cumulative Distribution Function

Example:

Suppose that $Y \sim B(n, p)$ with $n = 2$ and $p = 1/2$. Check whether $F(y)$ satisfy the properties of CDF.

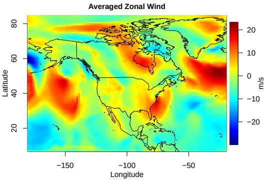
Solution:

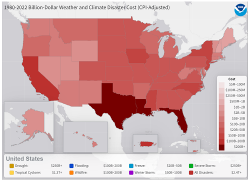
The CDF of Y is $F(y) = P(Y \le y) = \begin{cases} 0, & \text{for } y < 0, \\ 1/4, & \text{for } 0 \le y < 1, \\ 3/4, & \text{for } 1 \le y < 2, \\ 1, & \text{for } y \ge 2. \end{cases}$

1. $F(-\infty) = 0$.
2. $F(\infty) = 1$.
3. The plot of $F(y)$ clearly shows that $F(y)$ is nondecreasing.

Continuous Random Variables

- Not all random variables are discrete...
- Continuous random variables: are random variables that can take on
  - an infinite number of possible values.
  - any value in an interval.
- Continuous random variables can describe important real-world situations very well.

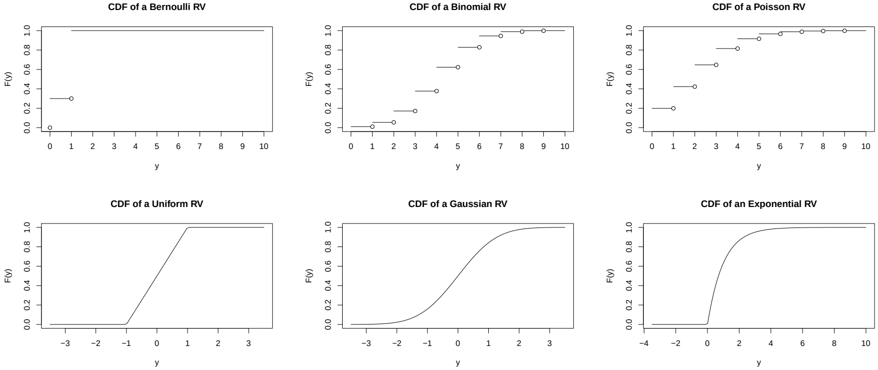




- Continuous mathematics (Calculus) is frequently easier to work with than mathematics of discrete random variables and distributions.

Definition 4.2: Continuous Random Variable

A *random variable* Y with CDF $F(y)$ is said to be *continuous* if $F(y)$ is continuous for $-\infty < y < \infty$.



Continuous Random Variables

Definition 4.3: CDF $\rightarrow$ PDF (Differentiation)

Let $F(y)$ be the CDF for a continuous random variable Y. Then $f(y)$, given by

$$f(y) = \frac{dF(y)}{dy} = F'(y)$$

wherever the derivative exists, is called the *probability density function* (PDF) for the random variable Y.

Note: PDF $\rightarrow$ CDF (Integration)

The CDF $F(y)$ can be retrieved from PDF $f(y)$ using the relationship:

$$F(y) = \int_{-\infty}^{y} f(t)dt,$$

where t is used as the variable of integration.

Theorem 4.2: Properties of a PDF

If $f(y)$ is a PDF for a continuous random variable, then

1. $f(y) \geq 0$ for all y, $-\infty < y < \infty$.
2. $\int_{-\infty}^{\infty} f(y)dy = 1$.

Some Remarks:

- PMF and PDF are NOT the same thing.
- In the discrete case, to find $P(Y = y)$, we evaluate the PMF at a single point y.
- The point mass for a continuous random variable Y is always equal to zero, i.e., $P(Y = y) = 0$.
- In the continuous case, we integrate the PDF over a certain interval to find the probability that Y will fall in that interval.
- $f(y)$ is NOT a probability.
- $f(y)$ is the height of the PDF when the random variable Y takes on the value y.
- $f(y)$ indicates how much probability is concentrated per unit length (dy) near y, or how dense the probability is near y.

Theorem 4.3

If the random variable Y has PDF $f(y)$ and $a < b$, then the probability that Y falls in the interval $[a, b]$ is

$$P(a \leq Y \leq b) = \int_a^b f(y)dy.$$

Note: The following four probabilities are equivalent for a continuous random variable Y:

$$P(a < Y < b) = P(a \leq Y < b) = P(a < Y \leq b) = P(a \leq Y \leq b).$$

This means that whether or not the endpoints of an interval are included, makes no difference when computing the probability of the interval.

Example 1:

Let X be a random variable with PDF $f(x) = 2x$ for $0 \leq x \leq 1$, and is 0 otherwise.

(a) Find the CDF.

(b) Find $P(\frac{1}{3} \leq X \leq \frac{1}{2})$.

Solution:

(a) $F(x) = \int_0^x 2xdx = x^2, \quad 0 \leq x \leq 1.$

(b) $P(\frac{1}{3} \leq X \leq \frac{1}{2}) = \int_{1/3}^{1/2} 2xdx = x^2\Big|_{1/3}^{1/2} = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{3}\right)^2 = \frac{9-4}{36} = \frac{5}{36}.$

Continuous Random Variables

Example 2:

The length of time to failure (in hundred of hours) for a transistor is a random variable Y with CDF

$$F(y) = \begin{cases} 0, & y < 0 \\ 1 - e^{-y^2}, & y \geq 0. \end{cases}$$

a. Show that $F(y)$ has the properties of a CDF.
b. Find the PDF $f(y)$.
c. Find the probability that the transistor operates for at least 200 hours.

Solution:

a. The properties of a CDF are satisfied since:
   - $\lim_{y \to -\infty} F(y) = \lim_{y \to -\infty} 0 = 0.$
   - $\lim_{y \to \infty} F(y) = \lim_{y \to \infty} (1 - e^{-y^2}) = 1.$
   - $F(y_2) - F(y_1) = e^{-y_1^2} - e^{-y_2^2} > 0$ if $y_1 < y_2$.

     (exp. function w/ negative exponent is monotone decreasing)

Example 2:

The length of time to failure (in hundred of hours) for a transistor is a random variable Y with CDF

$$F(y) = \begin{cases} 0, & y < 0 \\ 1 - e^{-y^2}, & y \geq 0. \end{cases}$$

a. Show that $F(y)$ has the properties of a CDF.
b. Find the PDF $f(y)$.
c. Find the probability that the transistor operates for at least 200 hours.

Solution:

b. Differentiating the CDF, we get the PDF

$$f(y) = F'(y) = \begin{cases} 0, & y < 0 \\ 2ye^{-y^2}, & y \geq 0. \end{cases}$$

Continuous Random Variables

Example 2:

The length of time to failure (in hundred of hours) for a transistor is a random variable Y with CDF

$$F(y) = \begin{cases} 0, & y < 0 \\ 1 - e^{-y^2}, & y \geq 0. \end{cases}$$

a. Show that $F(y)$ has the properties of a CDF.
b. Find the PDF $f(y)$.
c. Find the probability that the transistor operates for at least 200 hours.

Solution:

c. The probability that the transistor operates for at least 200 hours is

$$P(Y \geq 2) = 1 - P(Y < 2) = 1 - P(Y \leq 2),$$

since F is continuous at $y = 2$ implies that $P(Y = 2) = 0$.
Therefore,

$$P(Y \geq 2) = 1 - F(2) = 1 - (1 - e^{-4}) = e^{-4}.$$

Discrete vs. Continuous Random Variables

	Discrete	**Continuous**
Probabilistic Question	What is the probability that Y is equal to y? We assign probabilities to individual outcomes.	What is the probability of observing Y within the interval $[a, b]$? We assign probabilities to intervals of outcomes.
Values the r.v. can take	discrete points (countable)	continuous intervals (uncountable)
Probability Distribution	$p(y)$: probability mass function or PMF $0 \leq p(y) \leq 1$ $\sum_y p(y) = 1$	$f(y)$: probability density function or PDF $\frac{dF(y)}{dy} = f(y) \geq 0$ $\int_{-\infty}^{\infty} f(y)dy = 1$
Probability =	value of PMF at y	area under the PDF, bounded by a and b
Mean	$\mu = E(Y) = \sum_y yp(y)$	$\mu = E(Y) = \int_{-\infty}^{\infty} yp(y)$
Variance	$\sigma^2 = V(Y) = E\{(Y-\mu)^2\}$ $= \sum_y (y-\mu)^2 p(y)$	$\sigma^2 = V(Y) = E\{(Y-\mu)^2\}$ $= \int_{-\infty}^{\infty} (y-\mu)^2 p(y)$

Questions?

Homework Exercises: 4.5, 4.9, 4.13, 4.15, 4.17

Solutions will be discussed this Friday by the TA.