UCONN
UNIVERSITY OF CONNECTICUT

Department of Statistics
College of Liberal Arts & Sciences

STAT 3375Q: Introduction to Mathematical Statistics I

Spring 2024

Midterm 1 Exam Date: 14 February 2024

INSTRUCTIONS:

- There are 7 problems in this exam. Pick ONLY 5 problems to answer. Indicate your 5 chosen problems by circling the numbers on the table below. Answering more than 5 problems will NOT merit additional points. If you do not indicate your problem choices, I will check only the first five problems with written solutions.
- You are allowed ONE formula sheet which you will SUBMIT along with this exam sheet. Put all other items away such as books, notes, phones, laptops, and other electronic devices.
- You have 75 minutes to complete the exam. Time remaining will be flashed on the screen and will be updated every 10 minutes.
- A calculator is not necessary. You can keep your final answers as fractions in the simplest form.
- Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- To merit partial points, make sure to justify/explain your thoughts and solutions, using notations and terminologies properly, and clearly defining any events, random variables, parameters, and distributions that you used.
- Mysterious or unsupported answers will not receive full credit. A correct answer, unsupported by calculations, explanations, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Allocated Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
Total	100	

NAME: ______________________________

Problem 1

A pair of events A and B cannot be simultaneously mutually exclusive and independent. Prove that if $P(A) > 0$ and $P(B) > 0$, then:

a) If A and B are mutually exclusive, they cannot be independent. *(10 points)*

b) If A and B are independent, they cannot be mutually exclusive. *(10 points)*

Solution:

Problem 2
Prove each of the following statements. (Assume that any conditioning event has positive probability.)

a) If $P(B) = 1$, then $P(A|B) = P(A)$ for any A. *(5 points)*

b) If $A \subset B$, then $P(B|A) = 1$ and $P(A|B) = \frac{P(A)}{P(B)}$. *(5 points)*

c) If A and B are mutually exclusive, then

$$P(A|A \cup B) = \frac{P(A)}{P(A) + P(B)}.$$ *(5 points)*

d) $P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$. *(5 points)*

Solution:

Problem 3

Cards are dealt, one at a time, from a standard 52-card deck.

a) If the first 2 cards are both spades, what is the probability that the next 3 cards are also spades? *(6 points)*

b) If the first 3 cards are all spades, what is the probability that the next 2 cards are also spades? *(7 points)*

c) If the first 4 cards are all spades, what is the probability that the next card is also a spade? *(7 points)*

Solution:

Problem 4

Suppose that 5% of men and 0.25% of women are color blind. A person is chosen at random and that person is color blind. What is the probability that the person is male? (Assume males and females to be in equal numbers.) *(20 points)*

Solution:

Problem 5
Suppose X has the geometric PMF $p(x) = \frac{1}{3}\left(\frac{2}{3}\right)^x$, $x = 0, 1, 2, \ldots$. Determine the probability distribution of $Y = \frac{X}{X+1}$. Note that both X and Y are discrete random variables. To specify the probability distribution means to specify its PMF. *(20 points)*

Solution:

Problem 6

Suppose the random variable X has a Poisson distribution such that $P(X = 1) = P(X = 2)$.

a) Find $P(X = 4)$. *(10 points)*

b) Find $P(X \geq 4 | X \geq 2)$. *(10 points)*

Solution:

Problem 7

Let Y be the number of successes throughout n independent repetitions of a random experiment having the probability of success $p = \frac{1}{4}$. Determine the smallest value of n so that $P(Y \geq 1) \geq 0.70$. *(20 points)*

Solution: