UCONN
UNIVERSITY OF CONNECTICUT

Department of Statistics
College of Liberal Arts & Sciences

STAT 3375Q: Introduction to Mathematical Statistics I

Spring 2024

Midterm 1 Simulation Date: 12 February 2024

INSTRUCTIONS:

- There are 7 problems in this exam. Pick ONLY 5 problems to answer. Indicate your 5 chosen problems by circling the numbers on the table below. Answering more than 5 problems will NOT merit additional points.
- You are allowed ONE formula sheet which you will SUBMIT along with this exam sheet. Put all other items away such as books, notes, phones, laptops, and other electronic devices.
- You have 75 minutes to complete the exam. Time remaining will be flashed on the screen and will be updated every 10 minutes.
- A calculator is not necessary. You can keep your final answers as fractions in the simplest form.
- Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- To merit partial points, make sure to justify/explain your thoughts and solutions, using notations and terminologies properly, and clearly defining any events, random variables, parameters, and distributions that you used.
- Mysterious or unsupported answers will not receive full credit. A correct answer, unsupported by calculations, explanations, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Allocated Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
Total	100	

NAME: _____

Problem 1
Suppose $P(A) > 0$ and $P(B) > 0$. Prove that if $P(A|B) > P(A)$ then $P(B|A) > P(B)$. *(20 points)*

Solution:

Problem 2
Let $P(A) = 0.3$ and $P(B) = 0.6$.

a) Find $P(A \cup B)$ when A and B are independent. *(6 points)*

b) Find $P(A|\bar{B})$ when A and B are independent. *(7 points)*

c) Find $P(A|B)$ when A and B are mutually exclusive. *(7 points)*

Solution:

Problem 3
Five cards are dealt at random and without replacement from a standard deck of 52 cards. Note that there are 13 heart cards in the deck.

a) What is the probability that the hand contains 5 hearts? *(6 points)*

b) What is the probability that the hand contains at least 4 hearts? *(7 points)*

c) What is the probability that the hand contains 5 hearts if it is known that the hand contains at least 4 hearts? *(7 points)*

Solution:

Problem 4

Assume that an insurance company knows the following probabilities relating to automobile accidents (where the second column refers to the probability that the policyholder has at least one accident during the annual policy period):

Age of Driver	Probability of Accident	Fraction of Company's Insured Drivers
16-25	0.05	0.10
26-50	0.02	0.55
51-65	0.03	0.20
66-90	0.04	0.15

A randomly selected driver from the company's insured drivers has an accident. What is the probability that the driver is in the 16–25 age group? *(20 points)*

Solution:

Problem 5
A balanced die is thrown 5 times. Let X be the number of times that a number smaller than 3 had shown up.

a) Find the distribution function of X. *(10 points)*

b) Find the distribution function of $Y = 2X - 1$. *(10 points)*

Solution:

Problem 6
If the random variable X has a geometric distribution such that $V(X)/E(X) = a$, where $a > 0$.

a) What $E(X)$? Express your answer in terms of a. *(6 points)*

b) If $a = 1$, what is $P(X > 4)$? *(7 points)*

c) If $a = 1$, what is $P(X > 7|X > 3)$? *(7 points)*

Solution:

Problem 7
Four busses carrying 148 students arrive at a school. The busses carry 40, 33, 25 and 50 students. A student is selected at random from among the 148. Let X be the number of students on the bus of the randomly selected student. A random driver (from among the four drivers) is selected at random. Let Y be the number of students on the bus of the randomly selected driver.

a) Compute $E(X)$. *(10 points)*

b) Compute $E(Y)$. *(10 points)*

Solution: