UCONN
UNIVERSITY OF CONNECTICUT

Department of Statistics
College of Liberal Arts & Sciences

STAT 3375Q: Introduction to Mathematical Statistics I

Spring 2024

Midterm 2 Exam Date: 3 April 2024

INSTRUCTIONS:

- There are 7 problems in this exam. Pick ONLY 5 problems to answer. Indicate your 5 chosen problems by circling the numbers on the table below. Answering more than 5 problems will NOT merit additional points.
- You are allowed ONE formula sheet which you will SUBMIT along with this exam sheet. Put all other items away such as books, notes, phones, laptops, and other electronic devices.
- You have 75 minutes to complete the exam. Time remaining will be flashed on the screen and will be updated every 10 minutes.
- A calculator is not necessary. You can keep your final answers as fractions in the simplest form.
- Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- To merit partial points, make sure to justify/explain your thoughts and solutions, using notations and terminologies properly, and clearly defining any events, random variables, parameters, and distributions that you used.
- Mysterious or unsupported answers will not receive full credit. A correct answer, unsupported by calculations, explanations, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Allocated Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
Total	100	

NAME: _____

Problem 1
Suppose X and Y are independent Gaussian random variables. That is, $X \sim \mathcal{N}(1, 4)$ and $Y \sim \mathcal{N}(0, 7)$.

a) Find $\text{Cov}(X, Y)$. *(4 points)*

b) Find $E(X^2Y^2)$. *(4 points)*

c) Find $E(3X - 2Y)$. *(4 points)*

d) Find $V(3X - 2Y)$. *(4 points)*

e) Find $P(-3 \leq 3X - 2Y \leq 5)$. *(4 points)*
Hint: Sum of 2 Gaussian RVs is a Gaussian RV.

Solution:

Problem 2
Consider a random variable X with the PDF

$$f(x) = A + Bx^2, \quad 0 \le x \le 2.$$

If $E(X) = 1/2$, find A and B. *(20 points)*

Solution:

Problem 3
Suppose that the completion time in hours T for the STAT 3375Q final exam follows a distribution with density

$$f(t) = \frac{2}{27}(t^2 + t), \quad 0 \leq t \leq 3.$$

What is the probability that a randomly chosen student finishes the exam during the first 30 minutes. *(20 points)*

Solution:

Problem 4
Given that X has MGF

$$m(t) = \frac{1}{6}e^{-2t} + \frac{1}{3}e^{-t} + \frac{1}{4}e^{t} + \frac{1}{4}e^{2t},$$

what is the probability that X is even. *(20 points)*

Solution:

Problem 5
Suppose X and Y are continuous random variables with joint PDF

$$f(x,y) = \begin{cases} 4xy, & \text{if } 0 \le x \le 1; 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a) Find the marginal PDF of X, $f(x)$, and Y, $f(y)$. *(5 points)*

b) Find the conditional PDF of Y given X, $f(y|x)$. *(5 points)*

c) Find $P(Y \le 3/4 | X = 1/2)$. *(5 points)*

d) Find $E(Y|X = x)$. *(5 points)*

Solution:

Problem 6
Let X be a random variable with MGF

$$m(t) = \begin{cases} \frac{e^t - e^{-t}}{2t}, & \text{if } t \neq 0 \\ 1, & \text{if } t = 0. \end{cases}$$

a) Give the distribution of X. *(10 points)*

b) Compute $E(X)$ and $V(X)$. *(10 points)*

Solution:

Problem 7
Let X and Y be random variables such that

$$E(X) = 1, \quad E(X^2) = 3, \quad E(XY) = -4, \quad E(Y) = 2, \quad V(Y) = 25.$$

a) Find $E(2X + Y)$. *(4 points)*

b) Find $E\{X(2X + Y)\}$. *(4 points)*

c) Find $\text{Cov}(X, 2X + Y)$. *(4 points)*

d) Find $V(2X + Y)$. *(4 points)*

e) Find $\text{Corr}(X, 2X + Y)$. *(4 points)*

Solution:

Extra Credit: Estimate your score for this midterm exam. If your score is within the range of ± 5 points of your guess, you will get 5 extra points.