Department of Statistics
College of Liberal Arts & Sciences

STAT 3375Q: Introduction to Mathematical Statistics I
Spring 2024

Midterm 2 Simulation   Exam Date: 3 April 2024

INSTRUCTIONS:

- There are 7 problems in this exam. Pick ONLY 5 problems to answer. Indicate your 5 chosen problems by circling the numbers on the table below. Answering more than 5 problems will NOT merit additional points.

- You are allowed ONE formula sheet which you will SUBMIT along with this exam sheet. Put all other items away such as books, notes, phones, laptops, and other electronic devices.

- You have 75 minutes to complete the exam. Time remaining will be flashed on the screen and will be updated every 10 minutes.

- A calculator is not necessary. You can keep your final answers as fractions in the simplest form.

- Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

- To merit partial points, make sure to justify/explain your thoughts and solutions, using notations and terminologies properly, and clearly defining any events, random variables, parameters, and distributions that you used.

- Mysterious or unsupported answers will not receive full credit. A correct answer, unsupported by calculations, explanations, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Allocated Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
Total	100	

NAME: _____

Problem 1

Suppose X and Y are independent standard normal random variables. That is, $X \sim \mathcal{N}(0, 1)$ and $Y \sim \mathcal{N}(0, 1)$.

    a) Find $\text{Cov}(X, Y)$. *(4 points)*

    b) Find $E(X^2 Y^2)$. *(4 points)*

    c) Find $E(3X - 4Y)$. *(4 points)*

    d) Find $V(3X - 4Y)$. *(4 points)*

    e) Find $P(-3 \leq 3X - 4Y \leq 5)$. *(4 points)*

Solution:

Problem 2
Consider a random variable Y with the PDF

$$f(y) = \frac{|y|}{5}, \quad -1 < y < 3.$$

Find $E(Y)$. *(20 points)*

Solution:

Problem 3
Suppose that the completion time in hours T for the STAT 3375Q final exam follows a distribution with density

$$f(t) = \frac{2}{27}(t^2 + t), \quad 0 \le t \le 3.$$

What is the probability that a randomly chosen student finishes the exam during the second hour of the exam. *(20 points)*

Solution:

Problem 4
Given that X has MGF

$$m(t) = \frac{1}{6}e^{-2t} + \frac{1}{3}e^{-t} + \frac{1}{4}e^{t} + \frac{1}{4}e^{2t},$$

find $P(|X| \leq 1)$. *(20 points)*

Solution:

Problem 5
Suppose X and Y are continuous random variables with joint PDF

$$f(x,y) = \begin{cases} 15x^2y, & \text{if } 0 \le x \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a) Find the marginal PDF of X, $f(x)$. *(5 points)*

b) Find the conditional PDF of Y given X, $f(y|x)$. *(5 points)*

c) Find $P(Y \le 1/2 | X = 1/4)$. *(5 points)*

d) Find $E(Y|X=x)$. *(5 points)*

Solution:

Problem 6
Let X be a random variable with MGF

$$m(t) = \left(1 - \frac{t}{2}\right)^{-2}, \quad |t| < 2.$$

a) Find $E(X)$. *(7 points)*

b) Find $E(X^2)$. *(7 points)*

c) Find $V(X)$. *(6 points)*

Solution:

Problem 7

Let X and Y be random variables such that

$$E(X) = 1, \quad V(X) = 1, \quad E(Y) = 2, \quad V(Y) = 2, \quad \text{Cov}(X, Y) = 1.$$

a) Find $E(X + 2Y)$. *(5 points)*

b) Find $E(XY)$. *(5 points)*

c) Find $V(X - 2Y + 1)$. *(5 points)*

d) Find $\text{Corr}(X, Y)$. *(5 points)*

Solution: