UCONN
UNIVERSITY OF CONNECTICUT

Department of Statistics
College of Liberal Arts & Sciences

STAT 3375Q: Introduction to Mathematical Statistics I

Spring 2024

Quiz 1 Review Exercises Quiz Date: 24 January, 2024

Problem 1
Let E, F,G be three events. Find expressions for the following events using notations for intersection, union, and complement:

a. only F occurs,

b. both E and F but not G occur,

c. at least one event occurs,

d. at least two events occur,

e. all three events occur,

f. none occurs,

g. at most one occurs,

h. at most two occur.

Solution:

a. $F \cap \overline{E} \cap \overline{G}$.

b. $E \cap F \cap \overline{G}$

c. $E \cup F \cup G$

d. $(E \cap F) \cup (E \cap G) \cup (F \cap G)$

e. $E \cap F \cap G$

f. $\overline{(E \cup F \cup G)}$

g. $\overline{(E \cap F)} \cap \overline{(E \cap G)} \cap \overline{(F \cap G)}$

h. $\overline{E \cap F \cap G}$ (remove the event that E, F, and G happen)

□

Problem 2
If the occurrence of B makes A more likely, does the occurrence of A make B more likely?

Solution:

Given: $P(A|B) > P(A)$.

$\Rightarrow \frac{P(A \cap B)}{P(B)} > P(A)$ (conditional probability formula)

$\Rightarrow P(A \cap B) > P(B)P(A)$

$\Rightarrow \frac{P(A \cap B)}{P(A)} > P(B)$

$\Rightarrow P(B|A) > P(B)$ (conditional probability formula).

Yes, the occurrence of A make B more likely. $\square$

Problem 3
In a class there are four freshman boys, six freshman girls, and six sophomore boys. How many sophomore girls must be present if sex and class are to be independent when a student is selected at random?

Solution:

Given: 4 freshman boys, 6 freshman girls, 6 sophomore boys.

Let F be the event that the student is a freshman.

Let S be the event that the student is a sophomore.

Let B be the event that the student is a boy.

Let G be the event that the student is a girl.

Asked: The number of sophomore girls.

Let x be the number sophomore girls.

$P(F) = \frac{\text{num of freshmen}}{\text{total num of students}} = \frac{\text{4 boys+6 girls}}{\text{4 freshman boys+6 freshman girls+6 sophomore boys+}x} = \frac{10}{16+x}$.

$P(B) = \frac{\text{num of boys}}{\text{total num of students}} = \frac{\text{4 freshman boys+6 sophomore boys}}{\text{4 freshman boys+6 freshman girls+6 sophomore boys+}x} = \frac{10}{16+x}$.

$P(F \cap B) = \frac{\text{num of freshman boys}}{\text{total num of students}} = \frac{\text{4 freshman boys}}{\text{4 freshman boys+6 freshman girls+6 sophomore boys+}x} = \frac{4}{16+x}$.

We need to solve for x so that $P(F \cap B) = P(F)P(B)$.

$P(F \cap B) = P(F)P(B)$

$\Rightarrow \frac{4}{16+x} = \frac{10}{16+x}\frac{10}{16+x}$.

$\Rightarrow 16 + x = \frac{100}{4}$.

$\Rightarrow x = 9$.

There must be 9 sophomore girls. $\square$

Problem 4
Suppose we repeatedly roll two fair six-sided dice, considering the sum of the two values showing each time. What is the probability that the first time the sum is exactly 7 is on the third roll?

Solution:
The probability of getting 7 on any one role is $\frac{6}{36} = \frac{1}{6}$. Thus, the probability of not getting 7 on the first two roles, and then getting it on the third role, is equal to $\left(1 - \frac{1}{6}\right)\left(1 - \frac{1}{6}\right) = \frac{25}{216}$.

Problem 5
Suppose that we ask randomly selected people whether they share your birthday.

a. Give an expression for the probability that no one shares your birthday (ignore leap years).

b. How many people do we need to select so that the probability is at least 0.5 that at least one person shares your birthday?

Solution:

a. $\frac{(364)(364)\cdots(364)}{365^n} = \frac{364^n}{365^n}$.

a. With $n = 253$, $1 - \left(\frac{364}{365}\right)^{253} = 0.5005$.