STAT 3375Q: Introduction to Mathematical Statistics I
Spring 2024

Quiz 5 Review Exercises   Quiz Date: 17 April 2024

INSTRUCTIONS:

- The quiz is consist of only one problem taken from this list of five problems.

- This is a closed book, closed notes, closed laptop/computer quiz.

- The duration of the quiz is 15 minutes.

- The material for the quiz is Lectures 18-20 and the homework exercises.

- A calculator is not necessary. You can keep your final answers as fractions in the simplest form.

- To merit partial points, make sure to justify/explain your thoughts and solutions, using notation and terminology properly, and clearly defining any events and random variables that you use.

- Do not be late. The quiz will start at exactly 4:40pm and end at 4:55pm.

Problem 1
Let the random variable X have PDF $f(x) = \frac{30}{4}x^2(1-x)^2$ for $0 \leq x \leq 1$. Find the PDF of $Y = X^2$ using the Jacobian method.

Solution:

Problem 2
Find a transformation $G(U)$ such that if U has a uniform distribution on $(0, 1)$, then $G(U)$ has a uniform distribution on $(2, 4)$.

Solution:

Problem 3

Let X_1 and X_2 have the joint PDF

$$f_{X_1,X_2}(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)}, & \text{for } x_1 \geq 0, x_2 \geq 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Consider two RVs U_1 and U_2 defined in the following manner:

$$U_1 = X_1 + X_2 \quad \text{and} \quad U_2 = \frac{X_1}{X_1 + X_2}.$$

Find the joint PDF of U_1 and U_2.

Solution:

Problem 4
Let $Y_1, Y_2, \ldots, Y_{50} \sim \mathcal{U}(0,1)$. Find the mean and variance of the maximum RV, i.e., $E\{Y_{(50)}\}$ and $V\{Y_{(50)}\}$.

Solution:

Problem 5
Let $X_1, X_2, X_3, X_4 \overset{iid}{\sim} \mathcal{U}(0,5)$. Find the PDF of $X_{(3)}$.

Solution: