UCONN
UNIVERSITY OF CONNECTICUT

Department of Statistics
College of Liberal Arts & Sciences

STAT 3375Q: Introduction to Mathematical Statistics I

Spring 2024

Week 10 Homework Exercises Discussion Date: 29 March 2024

Problem 5.21

In Exercise 5.3, we determined that the joint probability distribution of Y_1, the number of married executives, and Y_2, the number of never-married executives, is given by

$$p(y_1, y_2) = \frac{\binom{4}{y_1}\binom{3}{y_2}\binom{2}{3-y_1-y_2}}{\binom{9}{3}},$$

where y_1 and y_2 are integers, $0 \leq y_1 \leq 3$, $0 \leq y_2 \leq 3$, and $1 \leq y_1 + y_2 \leq 3$.

a) Find the marginal probability of Y_1, the number of married executives among the three selected for promotion.

b) Find $P(Y_1 = 1|Y_2 = 2)$.

c) If we let Y_3 denote the number of divorced executives among the three selected for promotion, then $Y_3 = 3 - Y_1 - Y_2$. Find $P(Y_3 = 1|Y_2 = 1)$.

d) Compare the marginal distribution derived in (a) with the hypergeometric distributions with $N = 9$, $n = 3$, and $r = 4$ encountered in Section 3.7.

Solution:

Problem 5.23
In Example 5.4 and Exercise 5.5, we considered the joint density of Y_1, the proportion of the capacity of the tank that is stocked at the beginning of the week, and Y_2, the proportion of the capacity sold during the week, given by

$$f(y_1, y_2) = \begin{cases} 3y_1, & 0 \leq y_2 \leq y_1 \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

a) Find the marginal density function for Y_2.

b) For what values of y_2 is the conditional density $f(y_1|y_2)$ defined?

c) What is the probability that more than half a tank is sold given that three-fourths of a tank is stocked?

Solution:

Problem 5.27
In Exercise 5.9, we determined that

$$f(y_1, y_2) = \begin{cases} 6(1 - y_2), & 0 \leq y_1 \leq y_2 \leq 1, \\ 0, & \text{otherwise}, \end{cases}$$

is a valid joint probability density function. Find

a) the marginal density functions for Y_1 and Y_2.

b) $P(Y_2 \leq 1/2 | Y_1 \leq 3/4)$.

c) the conditional density function of Y_1 given $Y_2 = y_2$.

d) the conditional density function of Y_2 given $Y_1 = y_1$.

e) $P(Y_2 \geq 3/4 | Y_1 = 1/2)$.

Solution:

Problem 5.35
Refer to Exercise 5.33. If two minutes elapse between a customer's arrival at the store and his departure from the service window, find the probability that he waited in line less than one minute to reach the window.

Solution:

Problem 5.41
A quality control plan calls for randomly selecting three items from the daily production (assumed large) of a certain machine and observing the number of defectives. However, the proportion p of defectives produced by the machine varies from day to day and is assumed to have a uniform distribution on the interval $(0, 1)$. For a randomly chosen day, find the unconditional probability that exactly two defectives are observed in the sample.

Solution: