STAT 3375Q: Introduction to Mathematical Statistics I
Spring 2024

Week 12 Homework Exercises   Discussion Date: 12 Apr 2024

Problem 6.15

Let Y have a distribution function given by

$$F(y) = \begin{cases} 0, & y < 0, \\ 1 - e^{-y^2}, & y \geq 0. \end{cases}$$

Find a transformation $G(U)$ such that, if U has a uniform distribution on the interval $(0,1)$, $G(U)$ has the same distribution as Y.

Solution:

Problem 6.20
Let the random variable Y possess a uniform distribution on the interval $(0, 1)$. Derive the

a) distribution of $W = Y^2$.

b) distribution of $W = \sqrt{Y}$.

Solution:

Problem 6.23
Consider a random variable Y with PDF

$$f_Y(y) = \begin{cases} 2\,(1-y), & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Use the method of Jacobian transformation to find the PDFs of

a) $U_1 = 2Y - 1$.

b) $U_2 = 1 - 2Y$.

c) $U_3 = Y^2$.

Solution:

Problem 6.28
Let Y have a uniform $(0, 1)$ distribution. Show that $U = -2 \ln(Y)$ has an exponential distribution with mean 2.

Solution:

Problem 6.46
Suppose that Y has a Gamma distribution with $\alpha = n/2$ for some positive integer n and β equal to some specified value. Use the method of MGFs to show that $W = 2Y/\beta$ has a χ^2 distribution with n degrees of freedom.

Solution: