

**STAT 3375Q: Introduction to Mathematical Statistics I**  
Spring 2024

Week 13 Homework Exercises

Discussion Date: 19 April 2024

**Problem 6.72**

Let  $Y_1$  and  $Y_2$  be independent and uniformly distributed over the interval  $(0, 1)$ . Find

- a) the PDF of  $U_1 = \min(Y_1, Y_2)$ .
- b)  $E(U_1)$  and  $V(U_1)$ .

*Solution:*

**Problem 6.73**

Let  $Y_1$  and  $Y_2$  be independent and uniformly distributed over the interval  $(0, 1)$ . Find

- a) the PDF of  $U_2 = \max(Y_1, Y_2)$ .
- b)  $E(U_2)$  and  $V(U_2)$ .

*Solution:*

**Problem 6.74**

Let  $Y_1, Y_2, \dots, Y_n$  be independent, uniformly distributed random variables on the interval  $[0, \theta]$ . Find the

- a) CDF of  $Y_{(n)} = \max(Y_1, Y_2, \dots, Y_n)$ .
- b) PDF of  $Y_{(n)}$ .
- c)  $E(Y_{(n)})$  and  $V(Y_{(n)})$ .

*Solution:*

**Problem 6.75**

Refer to Exercise 6.74. Suppose that the number of minutes that you need to wait for a bus is uniformly distributed on the interval  $[0, 15]$ . If you take the bus five times, what is the probability that your longest wait is less than 10 minutes?

*Solution:*

**Problem 6.80**

Let  $Y_1, Y_2, \dots, Y_n$  be independent random variables, each with a beta distribution, with  $\alpha = \beta = 2$ . Find

- a) CDF of  $Y_{(n)} = \max(Y_1, Y_2, \dots, Y_n)$ .
- b) PDF of  $Y_{(n)}$ .
- c)  $E(Y_{(n)})$  when  $n = 2$ .

*Solution:*