

STAT 3375Q: Introduction to Mathematical Statistics I
Spring 2024

Lecture 1 Homework Exercises

Discussion Date: 19 January, 2024

Problem 2.1

Suppose a family contains two children of different ages, and we are interested in the gender of these children. Let F denote that a child is female and M that the child is male and let a pair such as FM denote that the older child is female and the younger is male. There are four points in the set S of possible observations:

$$S = \{FF, FM, MF, MM\}.$$

Let A denote the subset of possibilities containing no males. Let B be the subset containing two males. Let C be the subset containing at least one male. List the elements of

- a) A ,
- b) B
- c) C
- d) $A \cap B$
- e) $A \cup B$
- f) $A \cap C$
- g) $A \cup C$
- h) $B \cap C$
- i) $B \cup C$
- j) $C \cap \bar{B}$

Solution:

Problem 2.3

Draw Venn diagrams to verify DeMorgan's laws. That is, for any two sets A and B ,

a) $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$, and

b) $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$.

Solution:

Problem 2.5

Refer to Exercise 2.4. Use the identities $A = A \cap S$ and $S = B \cup \bar{B}$ and a distributive law to prove that

- a) $A = (A \cap B) \cup (A \cap \bar{B})$
- b) If $B \subset A$, then $A = B \cup (A \cap \bar{B})$.
- c) Further, show that $(A \cap B)$ and $(A \cap \bar{B})$ are mutually exclusive and therefore that A is the union of two mutually exclusive sets, $(A \cap B)$ and $(A \cap \bar{B})$.
- d) Also show that B and $(A \cap \bar{B})$ are mutually exclusive if $B \subset A$, A is the union of two mutually exclusive sets, B and $(A \cap \bar{B})$.

Solution:

Problem 2.7

A group of five applicants for a pair of identical jobs consists of three men and two women. The employer is to select two of the five applicants for the jobs. Let S denote the set of all possible outcomes for the employer's selection. Let A denote the subset of outcomes corresponding to the selection of two men. Let B denote the subset of outcomes corresponding to the selection of at least one woman. Denote the different men and women by $M_1, M_2, M_3, W_1,$ and W_2 . List the outcomes of the following:

- a) A
- b) \bar{B}
- c) $A \cup B$
- d) $A \cap B$
- e) $A \cap \bar{B}$

Solution:

Problem 2.8

From a survey of 60 students attending a university, it was found that 9 were living off campus, 36 were undergraduates, and 3 were undergraduates living off campus. Find the number of these students who were

- a) undergraduates, were living off campus, or both.
- b) undergraduates living on campus.
- c) graduate students living on campus.

Solution: