Solution for Week1 Discussion Session

Banani Bera

2.1

Given that, there are four points in the set S of possible observations:
S = {FF, FM, MF, MM }.
Let A denote the subset of possibilities containing no males; i.e. A = {FF}
B, the subset containing two males; i.e. B = {MM}
And C, the subset containing at least one male; i.e. C = {MF, FM, MM }

Then, $A \cap B = \phi$; as set A and set B don't have any element in common.
$A \cup B = FF, MM$; as union of A, B means the elements present collectively in A or B.
$A \cap C = \phi$; as set A and set C don't have any element in common.
$A \cup C = FF, FM, MF, MM = S$; as union of A, C contains all the elements in S.
$B \cap C = \{MM\}$; as set B and set C have one common element i.e. MM.
$B \cup C = FM, MF, MM = C$
Now, to find $C \cap \bar{B}$, first we have to find $\bar{B}$.
So, $\bar{B}$; the complement of set B contains all the elements in the universal set (S) but not in set B.
$C \cap \bar{B} = \{MF, FM\}$.

2.3

Venn diagrams to verify DeMorgan's laws that is, for any two sets A and B, $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$ and $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$.
To verify $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$, we will sketch the Venn diagrams below:

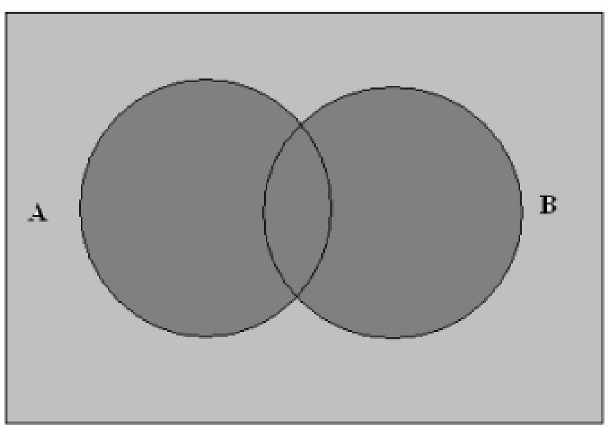

Figure 1: *Venn Diagram*

To verify $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$ again we draw the Venn diagrams:

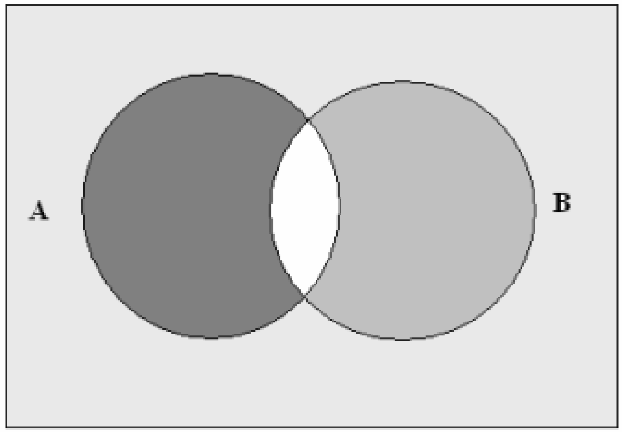

Figure 2: *Venn Diagram*

2.5

We are given the identities $A = A \cap S$ and $S = B \cup \bar{B}$.
(a) Note that $A = A \cap S = A \cap (B \cup \bar{B})$. Therefore, using the distributive law, we get $A = (A \cap B) \cup (A \cap \bar{B})$.
(b) If $B \subset A$, then $A \cap B = B$. Substituting $(A \cap B)$ by B in part (a) to get the result i.e. $A = (A \cap B) \cup (A \cap \bar{B}) = B \cup (A \cap \bar{B})$.
(c) $(A \cap B) \cap (A \cap \bar{B}) = A \cap (B \cap \bar{B}) \cap A = \phi$ (by associativity of intersection), since $B \cap \bar{B} = \phi$. Therefore, using part (a),$A = (A \cap B) \cup (A \cap \bar{B})$, with $(A \cap B)$ and $(A \cap \bar{B})$ mutually exclusive. This completes the proof.
(d) $B \cap (A \cap \bar{B}) = B \cap \bar{B} \cap A = \phi$. If $B \subset A$, then $A \cap B = B$. Substitute $(A \cap B)$ by B in part (c) to get the result.

2.7

From the given information we have,
A = {two males} = {(M1, M2), (M1,M3), (M2,M3)}
and B = {at least one female} = {(M1,W1), (M2,W1), (M3,W1), (M1,W2), (M2,W2), (M3,W2), (W1,W2)}
Now, $\bar{B} = \{nofemales\} = A$
$A \cup B = \{(M1, M2), (M1, M3), (M2, M3), (M1, W1), (M2, W1), (M3, W1), (M1, W2), (M2, W2), (M3, W2), (W1, W2)\} = S$
$A \cap B = \phi$
$A \cap \bar{B} = A$;as $\bar{B} = \{\text{no females}\} = A$.

2.8

Let consider two sets, one is undergrad student set and another is living off-campus students. From the given information we can draw the Venn Diagram as below:
Given, Total number of students = 60

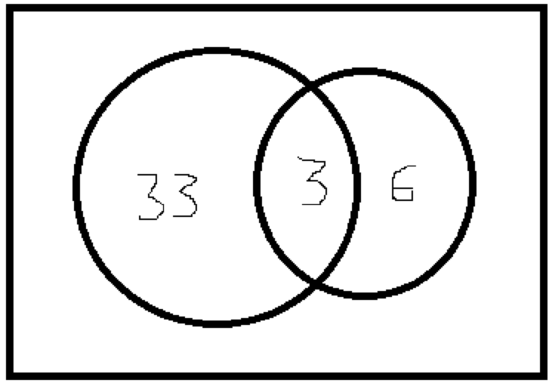

Figure 3: *Venn Diagram*

Number of Undergrad students = 36
Number of students living off campus = 9
And 3 were undergraduates living off campus i.e. undergrad $\cap$ living off-campus = 3.
(a) Number of undergraduates, were living off campus, or both i.e. undergrad $\cup$ living off-campus = (33+3+6) = 42.
(b) Number of undergraduates living on campus i.e. $\text{undergrad} \cap \overline{\text{living off-campus}}$ = undergrad - (undergrad $\cap$ living off-campus) = 36-3 = 33.
(c) Number of graduate students living on campus i.e. $\overline{\text{undergrad}} \cap \overline{\text{living off-campus}}$.
Using DeMorgan's Law, we can write,
$\overline{\text{undergrad}} \cap \overline{\text{living off-campus}} = \overline{(\text{undergrad} \cup \text{living off-campus})}$ = 60-42 = 18.