UCONN
UNIVERSITY OF CONNECTICUT

Department of Statistics
College of Liberal Arts & Sciences

STAT 3375Q: Introduction to Mathematical Statistics I

Spring 2024

Week 3 Homework Exercises Discussion Date: 2 February, 2024

Problem 3.7
Each of three balls are randomly placed into one of three bowls. Find the probability distribution for Y = the number of empty bowls.

Solution:

Problem 3.19
An insurance company issues a one-year \$1000 policy insuring against an occurrence A that historically happens to 2 out of every 100 owners of the policy. Administrative fees are \$15 per policy and are not part of the company's "profit." How much should the company charge for the policy if it requires that the expected profit per policy be \$50? (Hint: If C is the premium for the policy, the company's "profit" is $C - 15$ if does not occur and $C - 15 - 1000$ if A does occur.)

Solution:

Problem 3.27
A potential customer for an \$85,000 fire insurance policy possesses a home in an area that, according to experience, may sustain a total loss in a given year with probability of 0.001 and a 50% loss with probability 0.01. Ignoring all other partial losses, what premium should the insurance company charge for a yearly policy in order to break even on all \$85,000 policies in this area?

Solution:

Problem 3.31
Suppose that Y is a discrete random variable with mean μ and variance σ^2 and let $W = 2Y$.

a) Do you expect the mean of W to be larger than, smaller than, or equal to $\mu = E(Y)$? Why?

b) Use Theorem 3.4 to express $E(W) = E(2Y)$ in terms of $\mu = E(Y)$. Does this result agree with your answer to part a)?

c) Recalling that the variance is a measure of spread or dispersion, do you expect the variance of W to be larger than, smaller than, or equal to $\sigma^2 = V(Y)$? Why?

d) Use Definition 3.5 and the result in part b) to show that

$$V(W) = E[\{W - E(W)\}^2] = E\{4(Y - \mu)^2\} = 4\sigma^2;$$

That is, $W = 2Y$ has variance four times that of Y.

Solution:

Problem 3.33
Let Y be a discrete random variable with mean μ and variance σ^2. If a and b are constants, use Theorems 3.3, 3.4, 3.5, and 3.6 to prove that

a) $E(aY + b) = aE(Y) + b = a\mu + b$.

b) $V(aY + b) = a^2V(Y) = a^2\sigma^2$.

Solution: