Solutions for Week3 Discussion Session

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3.7

There are $3^3 = 27$ ways to place the three balls into the three bowls. Let Y be the number of empty bowls. So, Y can take only three values s.t. 0,1,2.
Then, $P(Y = 0) = P($ 0 bowls are empty$) = \frac{3!}{27} = \frac{6}{27}$
$P(Y = 2) = P($2 bowls are empty$) = \frac{3}{27}$
$P(Y = 1) = P($1 bowl is empty$) = 1 - \frac{3}{27} - \frac{6}{27} = \frac{18}{27}$

3.19

Let C is the premium for the policy, the company's "profit" is $(C - 15)$ if does not occur and $(C - 15 - 1000)$ if A does occur.
Hence, it is clear that A occurs 2 out of every 100 owners of the policy i.e. $P(A) = 0.02$ and A takes the profit value as (C-15-1000). Therefore (C-15) happens with probability 0.98 i.e. 98 out of 100 cases.
Given the expected profit per policy be \$50. That implies by the definition of expectation:
$E(\text{Profit}) = P(\text{A occurs}) * \text{profit value for occuring A} + P(\text{A does not occur}) * \text{profit value for not occuring A}$
$\implies 50 = (C\text{–}15)\frac{98}{100} + (C\text{–}15\text{–}1000)\frac{2}{100}$
$\implies C = 85$
The company should charge \$85 for the policy to make the expected profit.

3.27

Let Y = the payout on an individual policy. And, C represent the premium the insurance company charges. Then, the company's net gain/loss is given by C – Y. If E(C – Y) = 0, E(Y) = C.
$E(Y) = 0.001 * 85000 + 0.01 * 42500 + 0 * 0.989 = 510$
And we have E(Y) = C = 510.
The company should charge \$510.

3.31

(a) As $E(W) = E(2Y) = 2E(Y) = 2\mu$.
The mean of W will be larger than the mean of Y if $\mu > 0$. If $\mu < 0$, the mean of W will be smaller than μ. If $\mu = 0$, the mean of W will equal μ.
(b) According to theorem 3.4, $E[cg(Y)] = cE[g(Y)]$
Here our g(Y) = Y given. Hence, $E(2Y) = 2E(Y) = 2\mu$.
(c) The variance of W will be larger than σ^2, since the spread of values of W has increased.

(d)

$$\begin{aligned} V(W) &= E[WE(W)^2] \\ &= E[2Y - 2\mu^2] \\ &= E[4(Y - \mu)^2] \\ &= 4E[(Y - \mu)^2] \\ &= 4V(Y) \\ &= 4\sigma^2 \end{aligned}$$

3.33

Let Y be a discrete random variable with mean μ and variance σ^2. And a and b are constants.
(a)

$$\begin{aligned} E(aY + b) &= E(aY) + E(b); \quad \text{as Expectation is a linear operator} \\ &= aE(Y) + b; \quad \text{as} E(constant) = constant \\ &= a\mu + b \end{aligned}$$

(b)

$$\begin{aligned} V(aY + b) &= E[(aY + b) - E(aY + b)^2]; \quad \text{using theorem 3.6} \\ &= E[aY + b - aE(Y) - b^2]; \quad \text{as,} E(constant) = constant, \text{using 3.4} \\ &= E[a^2Y - \mu^2] \\ &= a^2V(Y) \\ &= a^2\sigma^2 \end{aligned}$$