UCONN
UNIVERSITY OF CONNECTICUT

Department of Statistics
College of Liberal Arts & Sciences

STAT 3375Q: Introduction to Mathematical Statistics I

Spring 2024

Week 4 Homework Exercises Discussion Date: 9 February, 2024

Problem 3.37

In 2003, the average combined SAT score (math and verbal) for college-bound students in the United States was 1026. Suppose that approximately 45% of all high school graduates took this test and that 100 high school graduates are randomly selected from among all high school grads in the United States. Which of the following random variables has a distribution that can be approximated by a binomial distribution? Whenever possible, give the values for n and p.

a) The number of students who took the SAT

b) The scores of the 100 students in the sample

c) The number of students in the sample who scored above average on the SAT

d) The amount of time required by each student to complete the SAT

e) The number of female high school grads in the sample

Solution:

Problem 3.43
Many utility companies promote energy conservation by offering discount rates to consumers who keep their energy usage below certain established subsidy standards. A recent EPA report notes that 70% of the island residents of Puerto Rico have reduced their electricity usage sufficiently to qualify for discounted rates. If five residential subscribers are randomly selected from San Juan, Puerto Rico, find the probability of each of the following events:

a) All five qualify for the favorable rates.

b) At least four qualify for the favorable rates.

Solution:

Problem 3.55
Suppose that Y is a binomial random variable with $n > 2$ trials and success probability p. Use the technique presented in Theorem 3.7 and the fact that $E\{Y(Y-1)(Y-2)\} = E(Y^3) - 3E(Y^2) + 2E(Y)$ to derive $E(Y^3)$.

Solution:

Problem 3.57
Refer to Exercise 3.56. Suppose the firm has a fixed cost of $20,000 in preparing equipment prior to doing its first exploration. If each successful exploration costs $30,000 and each unsuccessful exploration costs $15,000, find the expected total cost to the firm for its ten explorations.

Solution:

Problem 3.65
The maximum likelihood estimator for p is Y/n.

a) Derive $E(Y/n)$.

b) Derive $V(Y/n)$. What happens to $V(Y/n)$ as n gets large?

Solution: