

**STAT 3375Q: Introduction to Mathematical Statistics I**  
Spring 2024

Week 6 Homework Exercises

Discussion Date: 23 February 2024

**Problem 4.5**

Suppose that  $Y$  is a random variable that takes on only integer values  $1, 2, \dots$  and has distribution function  $F(y)$ . Show that the probability function  $p(y) = P(Y = y)$  is given by

$$p(y) = \begin{cases} F(1), & y = 1, \\ F(y) - F(y - 1), & y = 2, 3, \dots \end{cases}$$

*Solution:*

**Problem 4.9**

A random variable has the following distribution function:

$$F(y) = P(Y \leq y) = \begin{cases} 0, & y < 2, \\ 1/8, & 2 \leq y < 2.5, \\ 3/16, & 2.5 \leq y < 4, \\ 1/2, & 4 \leq y < 5.5, \\ 5/8, & 5.5 \leq y < 6, \\ 11/16, & 6 \leq y < 7, \\ 1, & y \geq 7. \end{cases}$$

- Is  $Y$  a continuous or discrete random variable? Why?
- What values of  $Y$  are assigned positive probabilities?
- Find the probability function for  $Y$ .
- What is the median,  $\phi_{0.5}$ , of  $Y$ ?

*Solution:*

**Problem 4.13**

A supplier of kerosene has a 150-gallon tank that is filled at the beginning of each week. His weekly demand shows a relative frequency behavior that increases steadily up to 100 gallons and then levels off between 100 and 150 gallons. If  $Y$  denotes weekly demand in hundreds of gallons, the relative frequency of demand can be modeled by

$$f(y) = \begin{cases} y, & 0 \leq y \leq 1, \\ 1, & 1 < y \leq 1.5, \\ 0, & \text{elsewhere.} \end{cases}$$

- a) Find  $F(y)$ .
- b) Find  $P(0 \leq Y \leq 0.5)$ .
- c) Find  $P(0.5 \leq Y \leq 1.2)$ .

*Solution:*

**Problem 4.15**

As a measure of intelligence, mice are timed when going through a maze to reach a reward of food. The time (in seconds) required for any mouse is a random variable  $Y$  with a density function given by

$$f(y) = \begin{cases} \frac{b}{y^2}, & y \geq b, \\ 0, & \text{elsewhere,} \end{cases}$$

where  $b$  is the minimum possible time needed to traverse the maze.

- a) Show that  $f(y)$  has the properties of a density function.
- b) Find  $F(y)$ .
- c) Find  $P(Y > b + c)$  for a positive constant  $c$ .
- d) If  $c$  and  $d$  are both positive constants such that  $d > c$ , find  $P(Y > b + d | Y > b + c)$ .

*Solution:*

**Problem 4.17**

The length of time required by students to complete a one-hour exam is a random variable with a density function given by

$$f(y) = \begin{cases} cy^2 + y, & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- a) Find  $c$ .
- b) Find  $F(y)$ .
- c) Graph  $f(y)$  and  $F(y)$ .
- d) Use  $F(y)$  in part (b) to find  $F(-1)$ ,  $F(0)$ , and  $F(1)$ .
- e) Find the probability that a randomly selected student will finish in less than half an hour.
- f) Given that a particular student needs at least 15 minutes to complete the exam, find the probability that she will require at least 30 minutes to finish.

*Solution:*