Solutions for Week7 Discussion Session

4.61

The random variable, let say X, follows a normal distribution with mean μ and standard deviation σ. The median of X will be μ. As we know that normal distribution is symmetric about μ. By the definition of symmetry, the mean = the median = the mode. Hence in this case, the median of a normally distributed random variable, X, is μ.

4.71

The wires produced by company A have a normal probability distribution with mean 0.13 ohm and standard deviation 0.005 ohm. So, let's denote that Y is the random variable follows normal probability distribution with mean 0.13 ohm and standard deviation 0.005 ohm.
Wires manufactured for use in a computer system are specified to have resistances between 0.12 and 0.14 ohms.

(a) The probability that a randomly selected wire from company A's production will meet the specifications means we have to find $P(0.12 < Y < 0.14)$. We will do that by standardizing Y
$P(0.12 < Y < 0.14) = P(\frac{0.12-0.13}{0.005} < Z < \frac{0.14-0.13}{0.005} = P(-2 < Z < 2) = 0.9544.$

(b) Let, X is a random variable denote the number of wires that meet specification. Here X follows a binomial with p = 0.9544 and we have taken four wires i.e. n=4.
Therefore, $P(X = 4) = (0.9544)^4 = 0.8297$

4.73

The width of bolts of fabric, say that is the random variable X is normally distributed with mean 950 mm (millimeters) and standard deviation 10 mm.

(a) The probability that a randomly chosen bolt has a width of between 947 and 958 mm is :$P(947 < X < 958) = P(\frac{947-950}{10} < X < \frac{958-950}{10}) = P(-0.3 < Z < 0.8)$
From the given standard normal table, we have, $P(Z > 0.8) = 0.2119, P(Z > -0.3) = 1 - 0.3821$
Hence, $P(-0.3 < Z < 0.8) = 1 - 0.3821 - 0.2119 = 0.406$

(b) We have to find the appropriate value for c such that a randomly chosen bolt has a width less than c with probability 0.8531 i.e. $P(X < c) = 0.8531 \implies P(Z < c') = 0.8531$taking z-transformation $\implies P(Z > c') = 1 - 0.8531 = 0.1469$. Now from the standard normal table we can find that probability 0.1469 is corresponding to the point 1.05. Hence our c' is 1.05 and then $c' = c - 950/10 \implies c = 960.5$

4.77

Given that SAT math scores, X have averaged 480 with standard deviation 100. And the average and standard deviation for ACT mathematics scores(say Y) are 18 and 6,respectively. As SAT and ACT college entrance exams are taken by thousands of students each year, hence we can approximate the distribution of X, Y as normal.

(a) For SAT the percentage of students will score below 550 in math in a typical year is: $P(X < 550 = P(Z < \frac{550-480}{100}) = P(Z < 0.7) = 0.758$; from the normal table.

(b) The score should the engineering school set as a comparable standard on the ACT math test i.e. the score $y' = 18 + (0.7 * 6) = 22.2$; as we need the same percentile as SAT score i.e. same z-score (0.7) in this case.

4.81

(a) Given the gamma function; $\Gamma(\alpha) = \int_0^\infty y^{\alpha-1}e^{-y}dy$
We will replace $\alpha = 1$ in the function. Hence, $\Gamma(1) = \int_0^\infty y^{1-1}e^{-y}dy = \int_0^\infty e^{-y}dy = -e^{-y}|_0^\infty = 1$

(b) $\Gamma(\alpha) = \int_0^\infty y^{\alpha-1}e^{-y}dy$
$= [-y^{\alpha-1}e^{-y}|_0^\infty] - \int_0^\infty(\alpha-1)y^{\alpha-2}[\int e^{-y}dy]dy$
$= 0 - \int_0^\infty(\alpha-1)y^{\alpha-2}[-e^{-y}]dy$
$= \int_0^\infty(\alpha-1)y^{\alpha-2}e^{-y}dy$
$= (\alpha-1)\Gamma(\alpha)$
Hence the proof.