UCONN
UNIVERSITY OF CONNECTICUT

Department of Statistics
College of Liberal Arts & Sciences

STAT 3375Q: Introduction to Mathematical Statistics I

Spring 2024

Week 8 Homework Exercises Discussion Date: 8 March 2024

Problem 4.93

Historical evidence indicates that times between fatal accidents on scheduled American domestic passenger flights have an approximately exponential distribution. Assume that the mean time between accidents is 44 days.

a) If one of the accidents occurred on July 1 of a randomly selected year in the study period, what is the probability that another accident occurred that same month?

b) What is the variance of the times between accidents?

Solution:

Problem 4.95
Let Y be an exponentially distributed random variable with mean β. Define a random variable X in the following way: $X = k$ if $k - 1 \leq Y < k$ for $k = 1, 2, \ldots$

a) Find $P(X = k)$ for each $k = 1, 2, \ldots$

b) Show that your answer to part (a) can be written as

$$P(X = k) = \left(e^{-1/\beta}\right)^{k-1} \left(1 - e^{-1/\beta}\right), \quad k = 1, 2, \ldots$$

and that X has a geometric distribution with $p = \left(1 - e^{-1/\beta}\right)$.

Solution:

Problem 4.101
Refer to Exercise 4.88. Suppose that the magnitude of earthquakes striking the region has a gamma distribution with $\alpha = 0.8$ and $\beta = 2.4$.

a) What is the mean magnitude of earthquakes striking the region?

b) What is the probability that the magnitude an earthquake striking the region will exceed 3.0 on the Richter scale?

c) Compare your answers to Exercise 4.88(a). Which probability is larger? Explain.

d) What is the probability that an earthquake striking the regions will fall between 2.0 and 3.0 on the Richter scale?

Solution:

Problem 4.103
Explosive devices used in mining operations produce nearly circular craters when detonated. The radii of these craters are exponentially distributed with mean 10 feet. Find the mean and variance of the areas produced by these explosive devices.

Solution:

Problem 4.111
Suppose that Y has a gamma distribution with parameters α and β.

a) If a is any positive or negative value such that $\alpha + a > 0$, show that

$$E(Y^a) = \frac{\beta^a \Gamma(\alpha + a)}{\Gamma(\alpha)}.$$

b) Why did your answer in part (a) require that $\alpha + a > 0$?

c) Show that, with $a = 1$, the result in part (a) gives $E(Y) = \alpha\beta$.

d) Use the result in part (a) to give an expression for $E(\sqrt{Y})$. What do you need to assume about α?

e) Use the result in part (a) to give an expression for $E(1/Y)$, $E(1/\sqrt{Y})$, and $E(1/Y^2)$. What do you need to assume about α in each case?

Solution: